

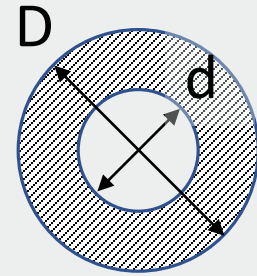
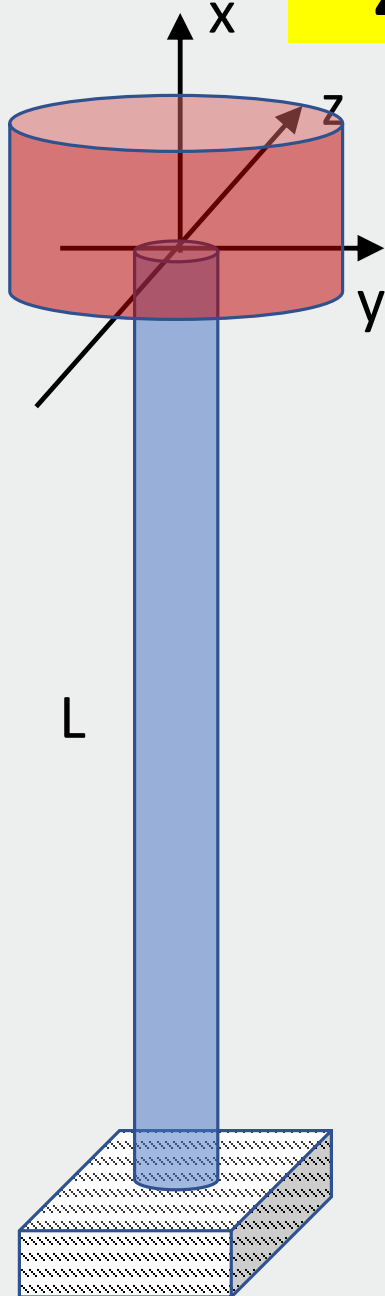
# *Wytrzymałość konstrukcji 1*

## Ćwiczenia 14

# Wyboczenie pręta

### Zad. 14.1

Q



$$D = 10 \text{ cm} , d = 4 \text{ cm} , L = 3 \text{ m}$$

$$E = 2 \cdot 10^5 \text{ MPa} , \sigma_{spr} = 220 \text{ MPa} , n_{kr} = 5$$

$Q_{max} = ?$

Smukłość graniczna:

$$\lambda_{gr} = \pi \sqrt{\frac{E}{\sigma_{spr}}} = \pi \sqrt{\frac{2 \cdot 10^5}{220}} = 95$$

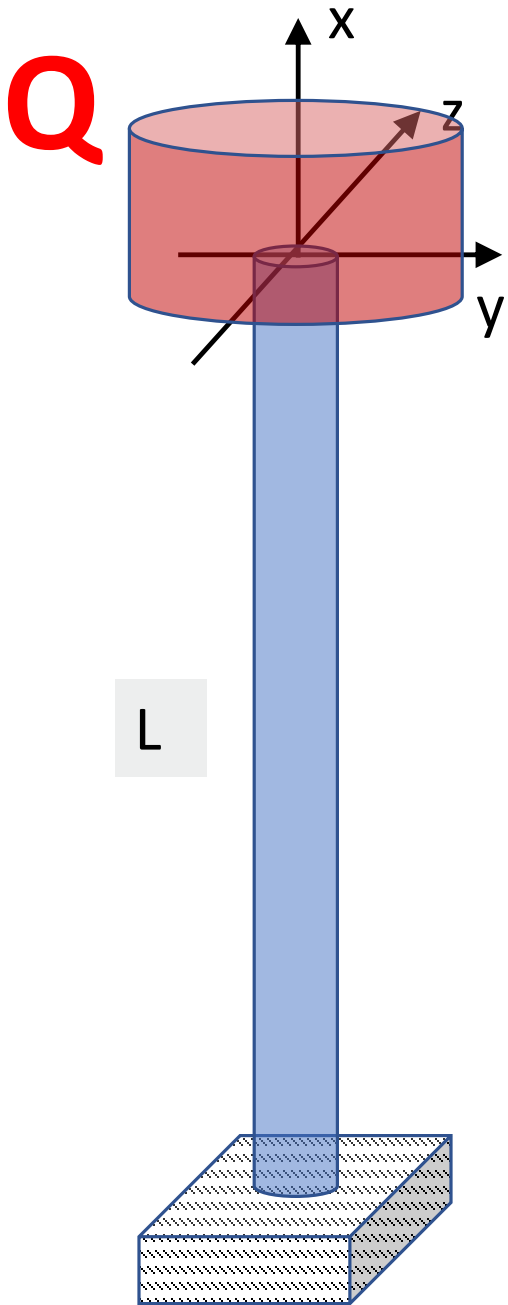
Charakterystyki przekroju:

$$J_y = \frac{\pi (D^4 - d^4)}{64} = 478 \text{ cm}^4$$

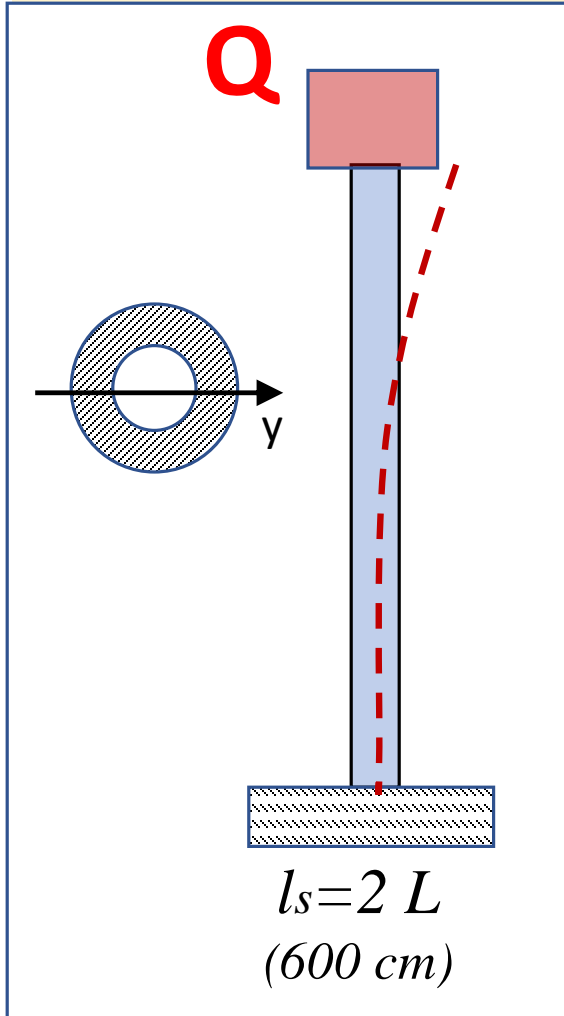
$$A = \frac{\pi (D^2 - d^2)}{4} = 66 \text{ cm}^2$$

Promień bezwładności:

$$i_y = \sqrt{\frac{J_y}{A}} = \sqrt{\frac{478}{66}} = 2.69 \text{ cm}$$



Określenie możliwej postaci wyboczenia:

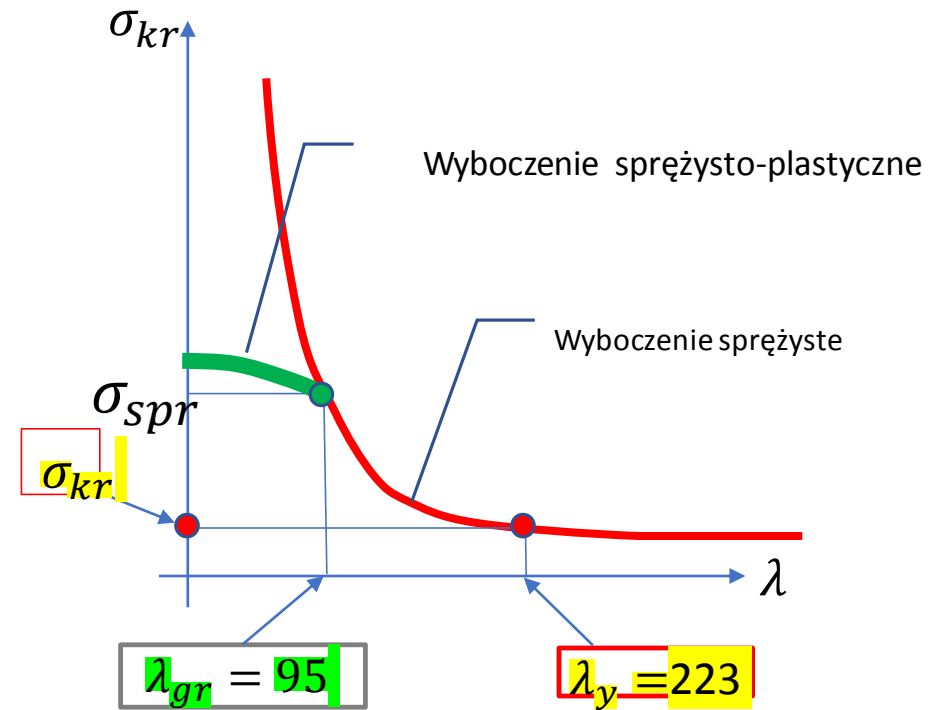


$$\lambda_{gr} = 95$$

$$i_y = 2.69 \text{ cm}$$

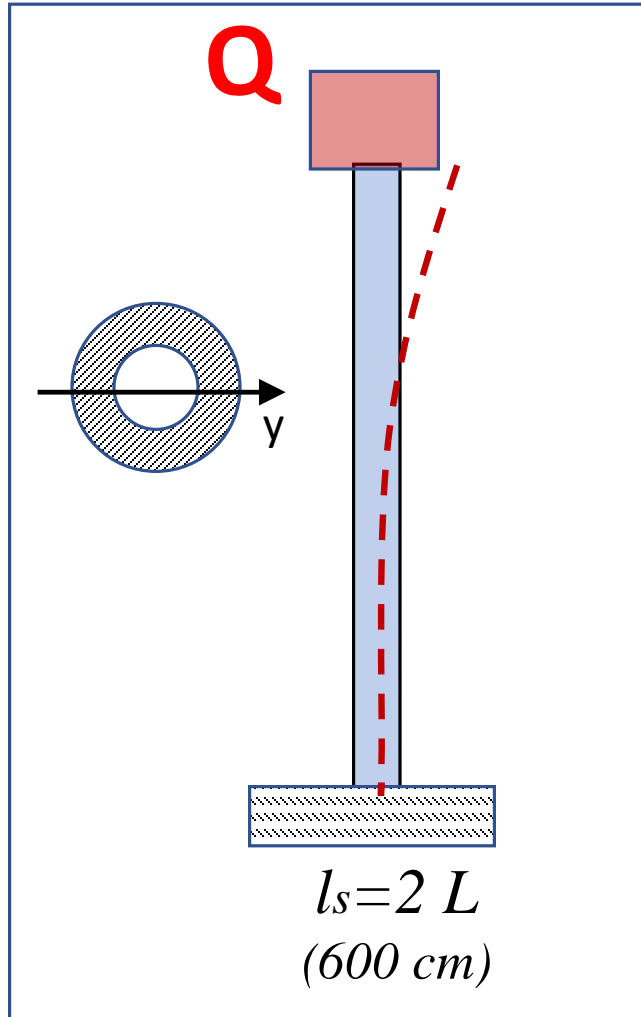
Określenie smukłości:

$$\lambda_y = \frac{l_s}{i_y} = \frac{600}{2.69} = 223$$



$$\lambda_y > \lambda_{gr}$$

Można stosować wzór Eulera



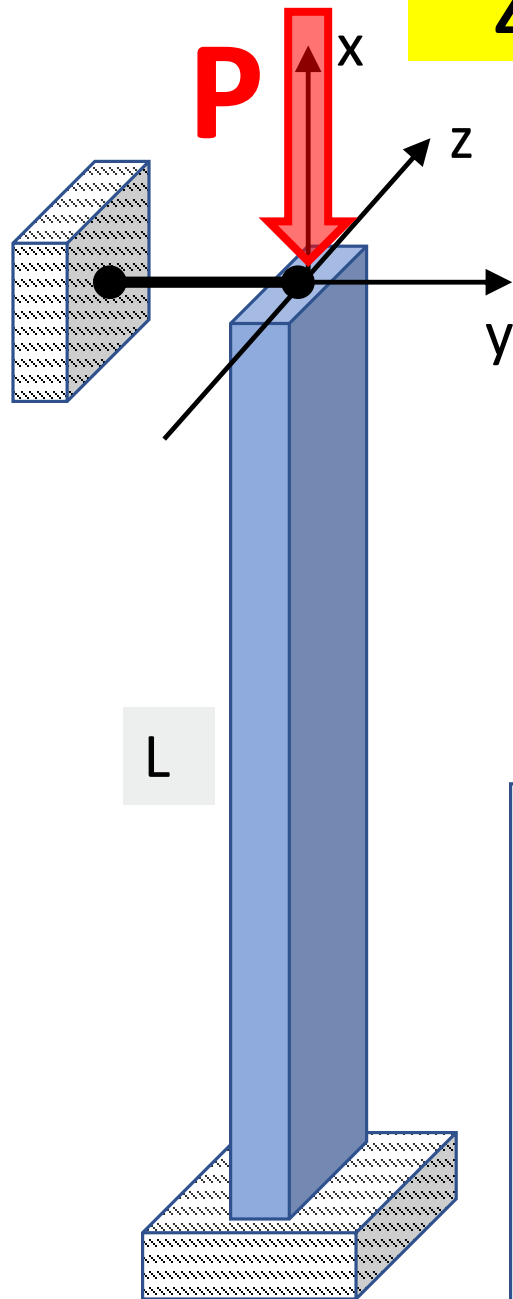
Określenie siły krytycznej i dopuszczalnej

$$P_{kr} = \frac{\pi^2 E J_y}{l_s^2} = \frac{\pi^2 \cdot 2 \cdot 10^{11} \cdot 478 \cdot 10^{-8}}{6^2} = 262093 \text{ N}$$

$$P_{dop} = \frac{P_{kr}}{n_{kr}} = \frac{262093}{5} = 52418 \text{ N}$$

$$Q_{max} \leq P_{dop} = 52.4 \text{ kN}$$

### Zad. 14.2



$H = 10 \text{ cm}$  ,  $B = 4 \text{ cm}$  ,  $L = 3 \text{ m}$   
 $E = 7 \cdot 10^4 \text{ MPa}$  ,  $R_{0,2} = 280 \text{ MPa}$  ,  $n_{kr} = 3$   
 $P_{max} = ?$

Smukłość graniczna:

$$\lambda_{gr} = \pi \sqrt{\frac{2E}{R_{0,2}}} = \pi \sqrt{\frac{2 \cdot 7 \cdot 10^4}{280}} = 70$$

Charakterystyki przekroju:

$$J_y = \frac{B \cdot H^3}{12} = \frac{4 \cdot 10^3}{12} = 333 \text{ cm}^4$$

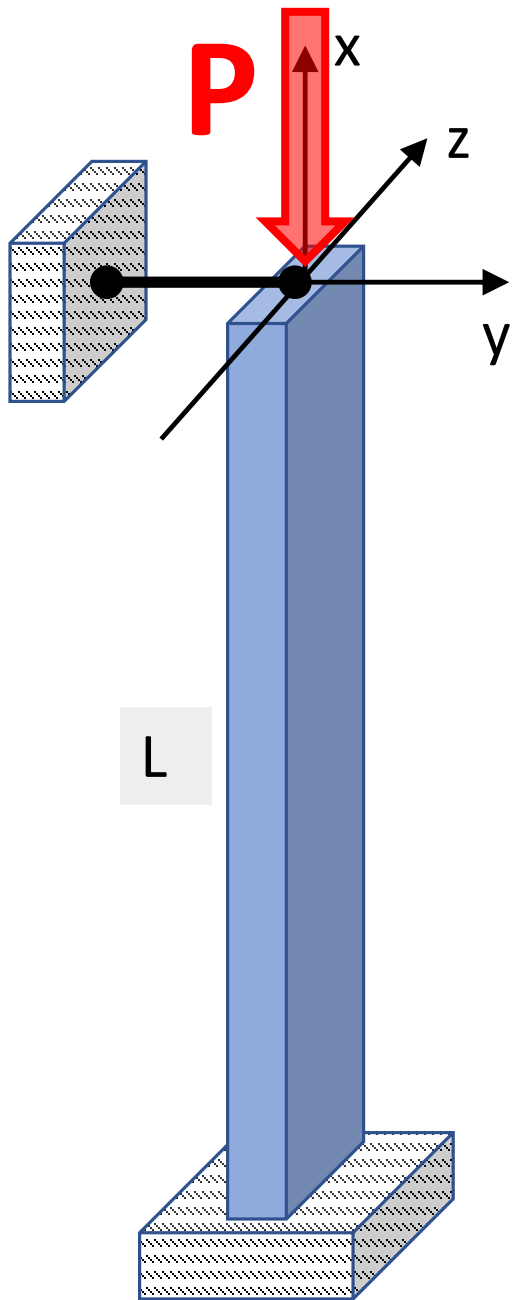
$$J_z = \frac{H \cdot B^3}{12} = \frac{10 \cdot 4^3}{12} = 53 \text{ cm}^4$$

$$A = B \cdot H = 4 \cdot 10 = 40 \text{ cm}^2$$

Promienie bezwładności:

$$i_y = \sqrt{\frac{J_y}{A}} = \sqrt{\frac{333}{40}} = 2.89 \text{ cm}$$

$$i_z = \sqrt{\frac{J_z}{A}} = \sqrt{\frac{53}{40}} = 1.15 \text{ cm}$$

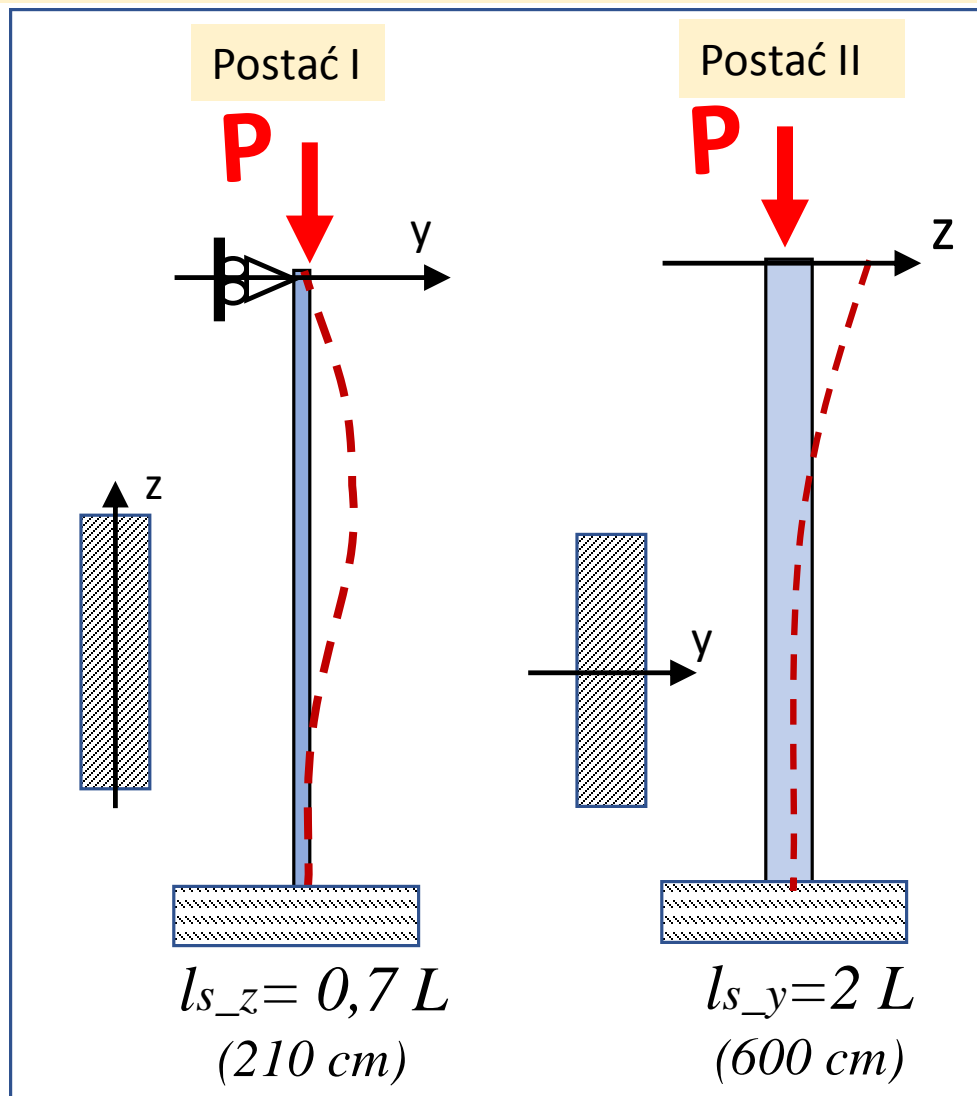


$$\lambda_{gr} = 70$$

$$i_y = 2.89 \text{ cm}$$

$$i_z = 1.15 \text{ cm}$$

Określenie możliwych postaci wybooczenia:



Określenie smukłości:

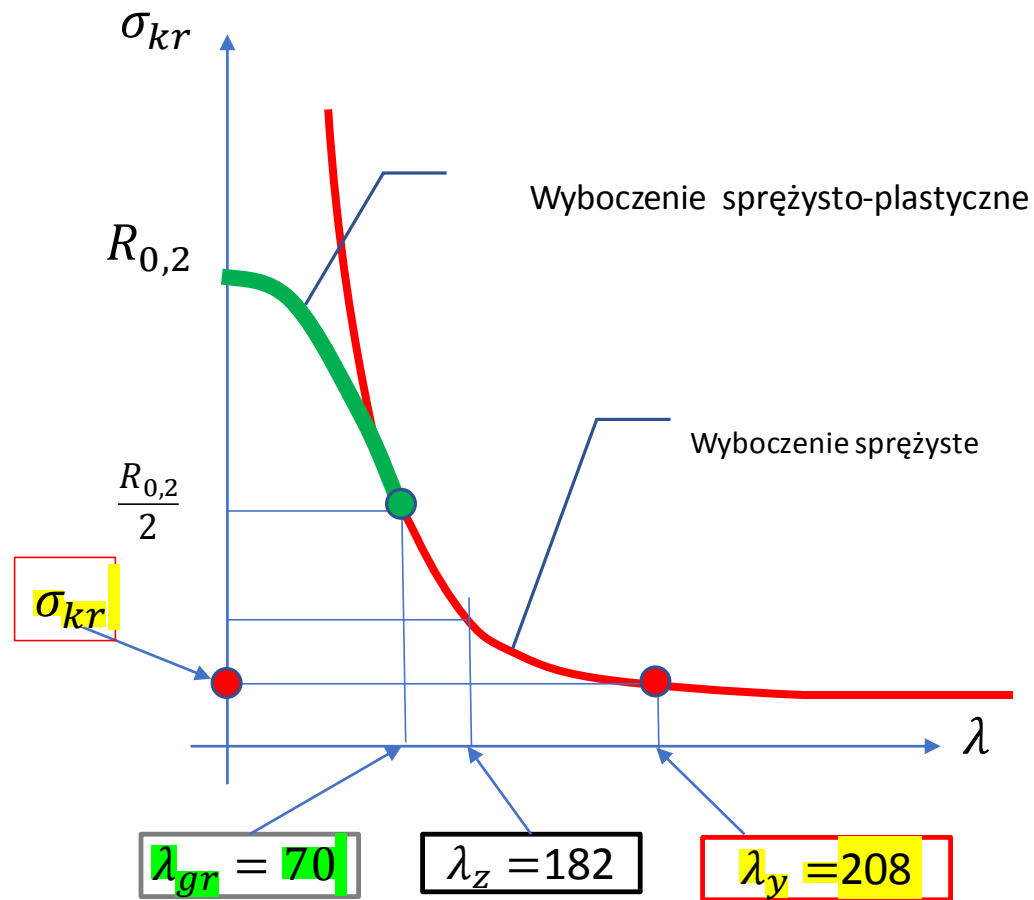
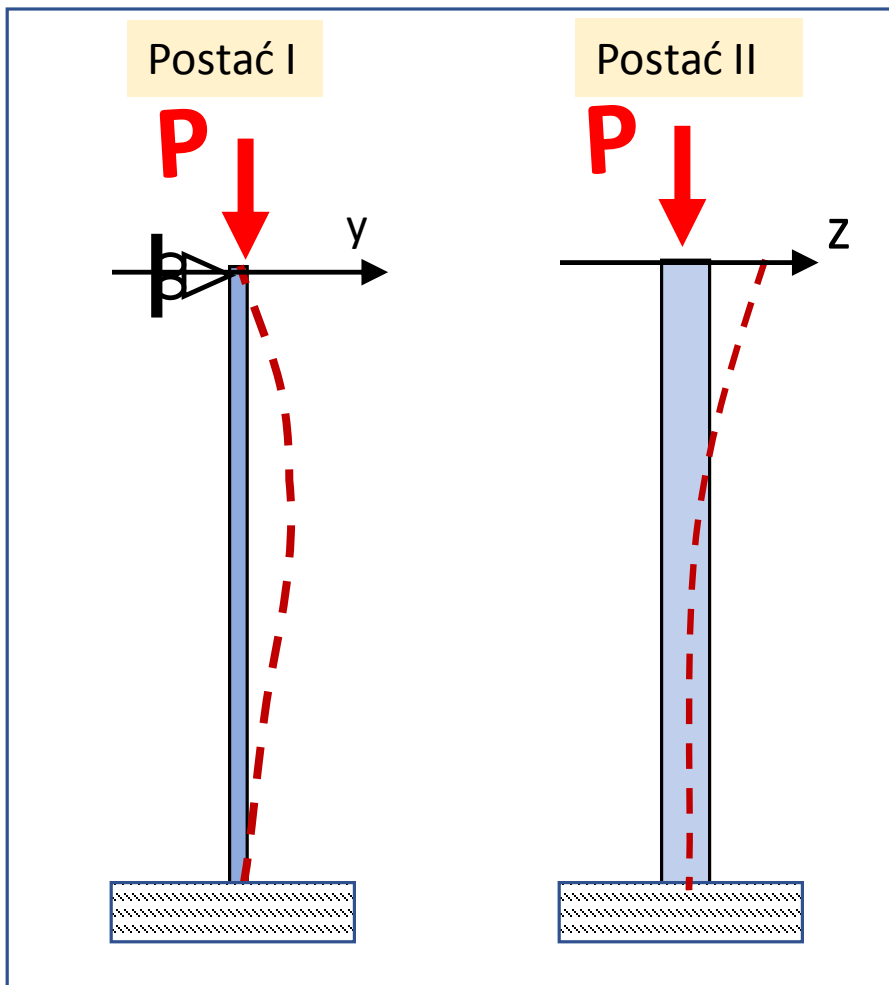
$$\lambda_y = \frac{l_{s_y}}{i_y} = \frac{600}{2.89} = 208$$

$$\lambda_z = \frac{l_{s_z}}{i_z} = \frac{210}{1.15} = 182$$

$$\lambda_y > \lambda_z > \lambda_{gr}$$

Można stosować wzór Eulera

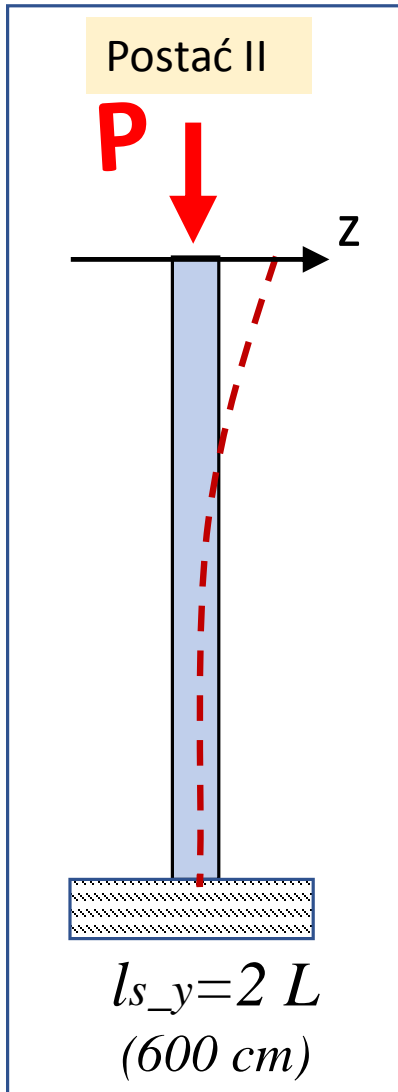
## Określenie postaci utraty stateczności



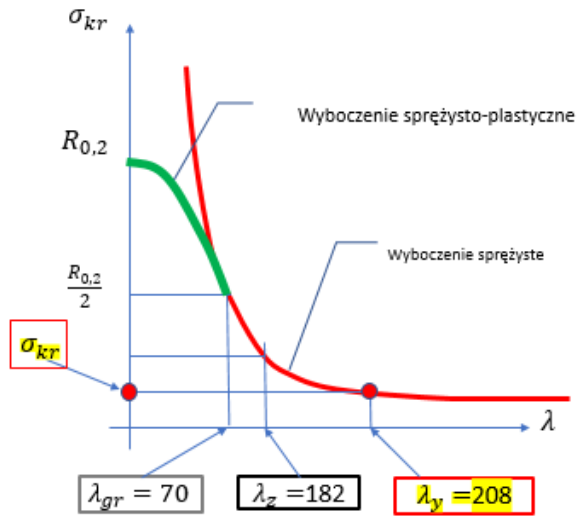
$\lambda_y > \lambda_z > \lambda_{gr}$  → Pręt utraci stateczność w postaci II

Można stosować wzór Eulera

Pręt utraci stateczność  
w postaci II



Określenie siły krytycznej i dopuszczalnej



$$P_{kr} = \frac{\pi^2 E J_y}{l_{s_y}^2} = \frac{\pi^2 7 \cdot 10^{10} \cdot 333 \cdot 10^{-8}}{6^2} = 63900 \text{ N}$$

$$P_{dop} = \frac{P_{kr}}{n_{kr}} = \frac{63900}{3} = 21301 \text{ N}$$

$$P_{max} \leq P_{dop} = 21.3 \text{ kN}$$



# Zad. 14.3

Kratownica płaska

Dane:

$P = 2000 \text{ N}$

$a = 1 \text{ m}$

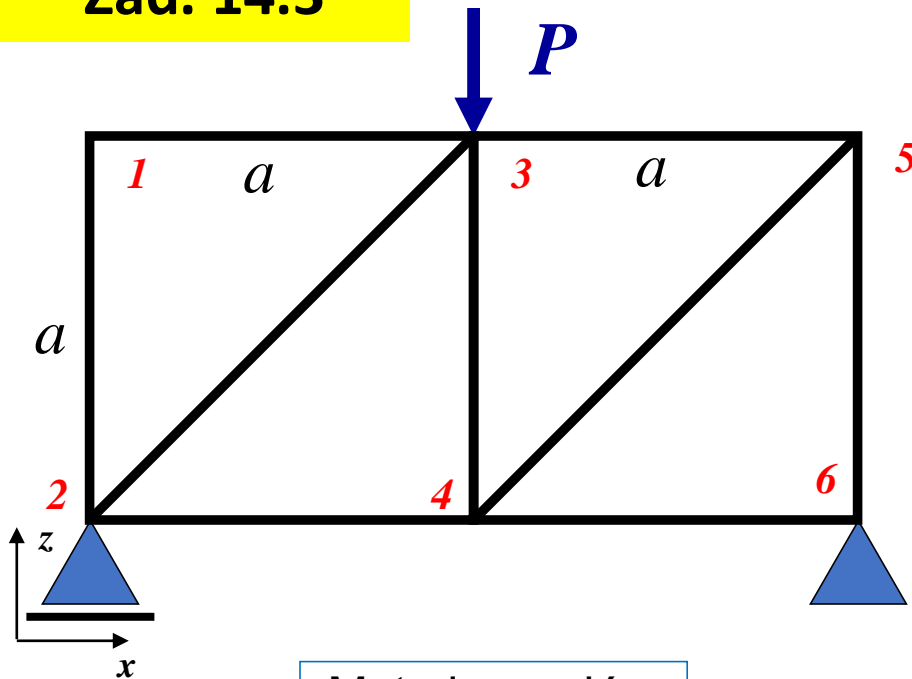
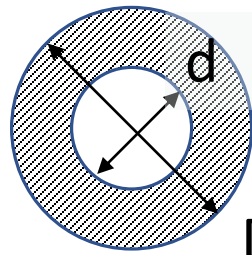
$E = 2 \cdot 10^5 \text{ MPa}$

$\lambda_{gr} = 100$

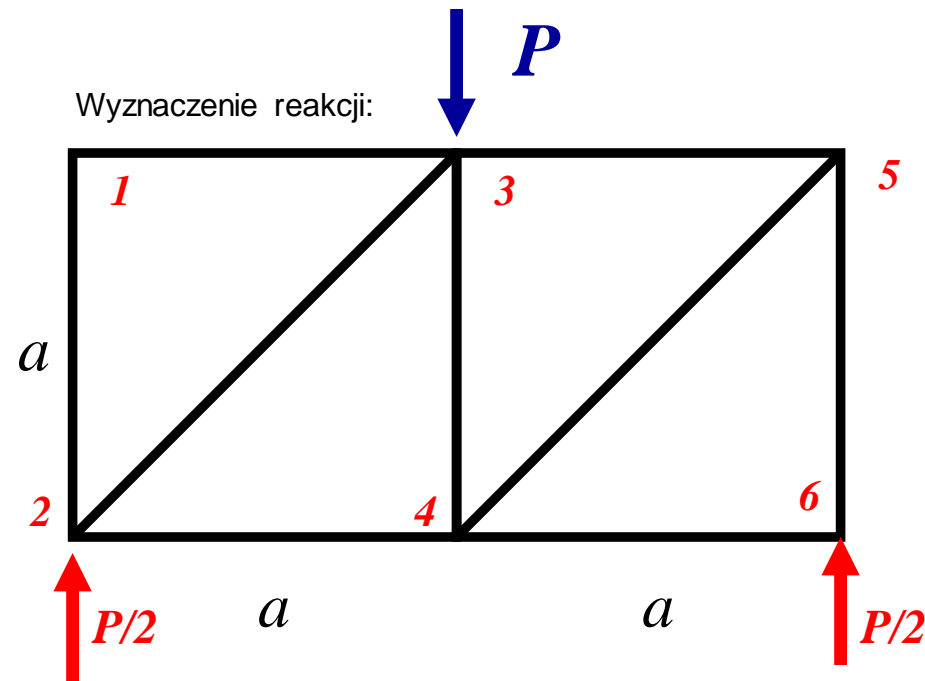
$n_{kr} = 5$

$D = 2d$

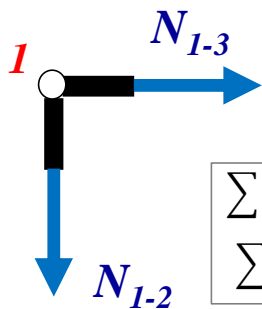
$d = ?$



Wyznaczenie reakcji:



Metoda węzłów

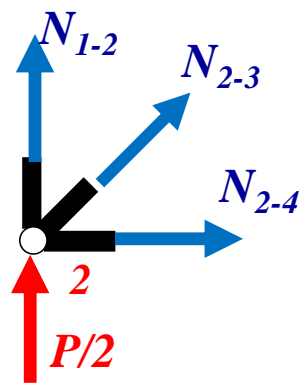


$$\sum F_x = 0$$

$$\sum F_z = 0$$

$$N_{1-3} = 0$$

$$N_{1-2} = 0$$

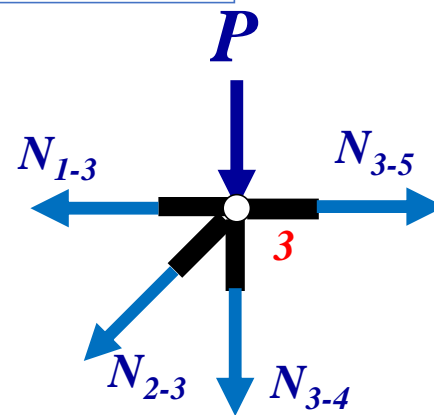


$$N_{2-4} + N_{2-3} \cos 45^\circ = 0$$

$$\frac{1}{2}P + N_{2-3} \sin 45^\circ = 0$$

$$N_{2-4} = \frac{1}{2}P$$

$$N_{2-3} = -\frac{1}{2}\sqrt{2}P$$



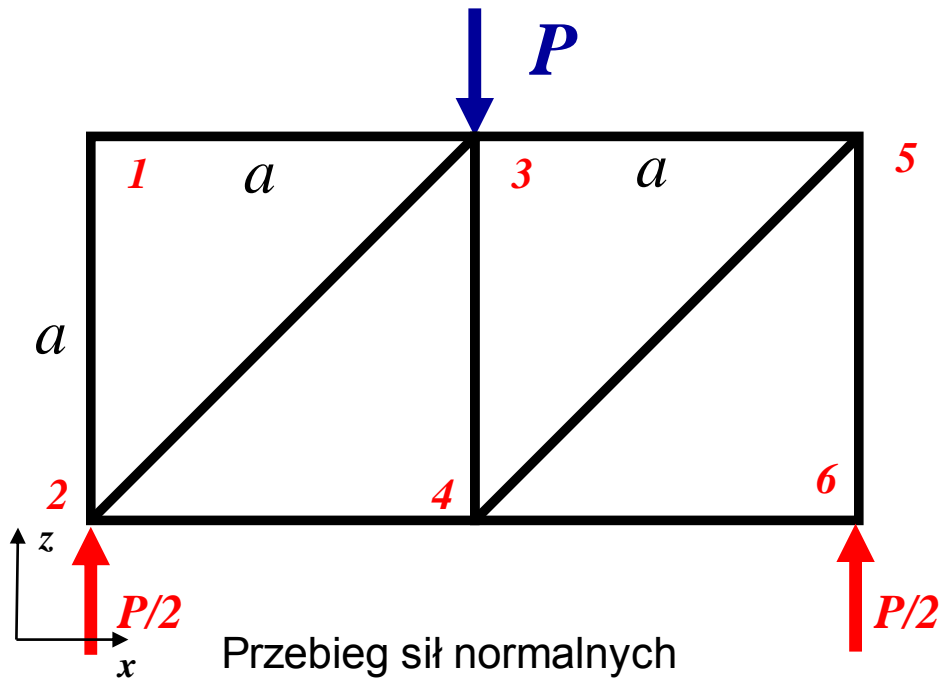
$$N_{3-5} - N_{2-3} \cos 45^\circ = 0$$

$$-P - N_{2-3} \sin 45^\circ - N_{3-4} = 0$$

$$N_{3-5} = -\frac{1}{2}P$$

$$N_{3-4} = -\frac{1}{2}P$$

i.t.d



$$N_{1-3}=0$$

$$N_{1-2}=0$$

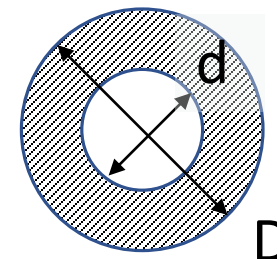
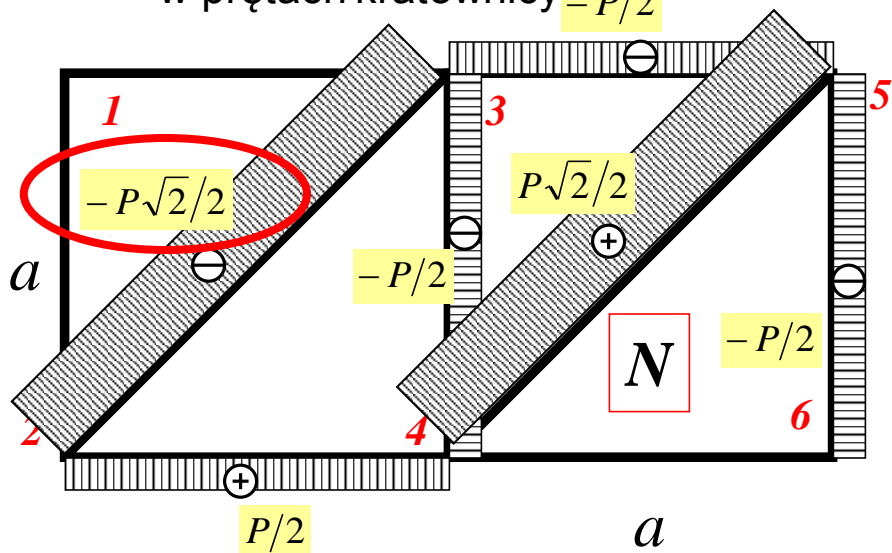
$$N_{2-4} = \frac{1}{2}P$$

$$N_{2-3} = -\frac{1}{2}\sqrt{2}P$$

$$N_{3-5} = -\frac{1}{2}P$$

$$N_{3-4} = -\frac{1}{2}P$$

i.t.d



Charakterystyki przekroju:

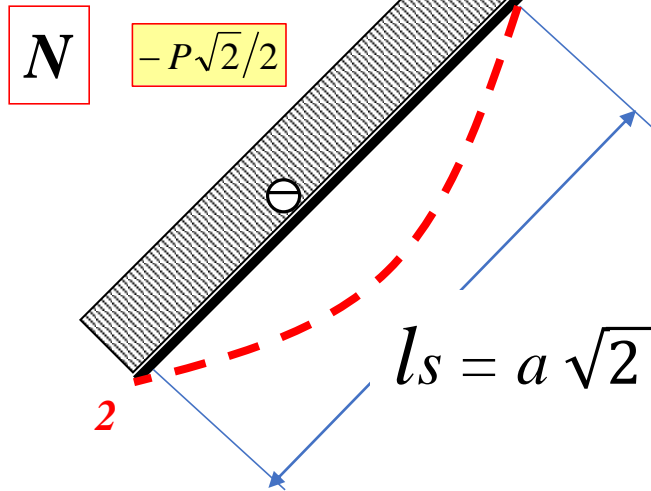
$$J_y = \frac{\pi (D^4 - d^4)}{64} = \frac{15 \pi d^4}{64}$$

$$A = \frac{\pi (D^2 - d^2)}{4} = \frac{3 \pi d^2}{4}$$

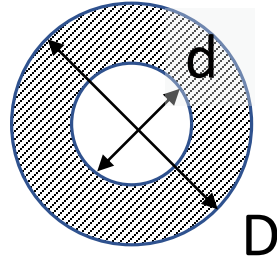
Promień bezwładności:

$$i_y = \sqrt{\frac{J_y}{A}} = \sqrt{\frac{15}{48}} d$$

Zagrożony ściskaniem pręt



Postać wyboczenia



$$J_y = \frac{15 \pi d^4}{64}$$

$$i_y = \sqrt{\frac{15}{48}} d$$

$$P_{kr} = \frac{\pi^2 E J_y}{l_s^2} = \frac{\pi^2 E \cdot \frac{15 \pi d^4}{64}}{(a \sqrt{2})^2}$$

$$P_{dop} = \frac{P_{kr}}{n_{kr}} = \frac{\pi^2 E \cdot \frac{15 \pi d^4}{64}}{n_{kr} 2 a^2} \geq P \frac{\sqrt{2}}{2}$$

$$d^4 \geq \frac{32 \sqrt{2} n_{kr} P 2 a^2}{15 \pi^3 E} = \frac{32 \sqrt{2} \cdot 5 \cdot 2000 \cdot 2 \cdot 1^2}{15 \pi^3 2 \cdot 10^{11}}$$

$$d \geq 0.00993 \text{ m} = 9.93 \text{ mm}$$



$$i_y = \sqrt{\frac{15}{48}} d = 5.55 \text{ mm}$$

Sprawdzenie smukłości:

$$\lambda_y = \frac{l_s}{i_y} = \frac{1000 \sqrt{2}}{5.55} = 254 \geq \lambda_{gr}$$

OK